Solution of a system of linear equation by matrix method.

Let us consider three linear equation x, y & z are -

$$a_1x+b_1y+c_1z=d_1$$
$$a_2x+b_2y+c_2z=d_2$$
$$a_3x+b_3y+c_3z=d_3.$$

Then the above equation can be written in matrix form as —

$$\begin{bmatrix} a_1x+b_1y+c_1z \\ a_2x+b_2y+c_2z \\ a_3x+b_3y+c_3z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$\Rightarrow AX = B. \quad \text{where } A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix},$$

This eqn is said to be the matrix equation of the given system of linear eqn.

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \; \& \; B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Imp Theorem :-> Prove that the matrix eqn. $AX = B$ has a unique solution if A is non-singular i.e, $|A| \neq 0$.

Soln:-> The given eqn is —

$$AX = B \quad \text{--(1)}$$

since $|A| \neq 0$, A^{-1} exists and

$$AA^{-1} = A^{-1}A = I.$$

Now, $AX = B$

$$\Rightarrow A^{-1}(AX) = A^{-1}B$$
$$\Rightarrow (A^{-1}A)X = A^{-1}B$$
$$\Rightarrow IX = A^{-1}B$$
$$\Rightarrow X = A^{-1}B$$

This shows that $X = A^{-1}B$ is a solution of the given eqn. **

Uniqueness:—

If possible, let X_1 and X_2 be two solutions of the eqn (1) Then,

$$AX_1 = B \ \& \ AX_2 = B.$$

$$\therefore AX_1 = AX_2$$

$$\Rightarrow A^{-1}(AX_1) = A^{-1}(AX_2)$$

$$\Rightarrow (A^{-1}A)X_1 = (A^{-1}A)X_2$$

$$\Rightarrow IX_1 = IX_2$$

$$\Rightarrow X_1 = X_2$$

This shows that the eqn (1) has a unique solution. #

Note:→ The eqn $AX = B$.

(i) has a unique solution if $|A| \neq 0$ and its solution is given by $X = A^{-1}B$.

(ii) has infinite no. of solution if $|A| = 0$ and $(adj\,A)B$ is a zero matrix.

(iii) has no solution if $|A| = 0$ and $(adj\,A)B$ is a non-zero matrix.

Q:— Solve by matrix method:—

07 (i)
$$x+y+z=6$$
$$x-y+2z=5$$
$$2x-2y+3z=7.$$

06 (ii)
$$3x+y+z=1$$
$$2x+5y+7z=52$$
$$2x-y+z=4.$$

** 05 (iii)
$$x+y+z=1$$
$$3x+4y+5z=2$$
$$2x+3y+4z=1$$

04 (iv)
$$x+y+z=3$$
$$2x+3y-4z=1$$
$$3x+5y-8z=0.$$

(v)
$$x+z=0$$
$$3x+4y+5z=2$$
$$2x+3y+4z=2.$$

(1) Soln:- The given system of equations are -

$$x+y+z=6$$
$$x-y+2z=5$$
$$2x-2y+3z=7$$

The above eqn can be written in matrix form as -

$$AX=B \quad \text{---(1)}$$

where $A=\begin{bmatrix}1&1&1\\1&-1&2\\2&-2&3\end{bmatrix}$, $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$, $B=\begin{bmatrix}6\\5\\7\end{bmatrix}$

Now,

$$|A|=\begin{vmatrix}1&1&1\\1&-1&2\\2&-2&3\end{vmatrix}$$
$$=1(-3+4)-1(3-4)+1(-2+2)$$
$$=1+1+0$$
$$=2$$
$$\neq 0.$$

Since $|A|\neq 0$, The given equation has unique soln.

Let us find A^{-1}.

1st row Co-factor of $1 = \begin{vmatrix}-1&2\\-2&3\end{vmatrix} = -3+4=1.$

" " $1 = -\begin{vmatrix}1&2\\2&3\end{vmatrix} = -(3-4)=1$

" " $1 = \begin{vmatrix}1&-1\\2&-2\end{vmatrix} = -2+2=0.$

2nd row

Co-factor of $1 = -\begin{vmatrix}1&1\\-2&3\end{vmatrix} = -(3+2)=-5$

" " $-1 = \begin{vmatrix}1&1\\2&3\end{vmatrix} = (3-2)=1$

" " $2 = -\begin{vmatrix}1&1\\2&-2\end{vmatrix} = -(-2-2)=4$

P.T.O.