

3rd row

$$\text{co-factor of } 2 = \begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} = 2 + 1 = 3$$

$$\text{" " } -2 = - \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = - (2 - 1) = -1$$

$$\text{" " } 3 = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1 - 1 = -2$$

$$\text{Co-factor matrix of } A = \begin{bmatrix} 1 & 1 & 0 \\ -5 & 1 & 4 \\ 3 & -1 & -2 \end{bmatrix}$$

$$\therefore \text{adj } A = \begin{bmatrix} 1 & -5 & 3 \\ 1 & 1 & -1 \\ 0 & 4 & -2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \cdot (\text{adj } A)$$

$$= \frac{1}{2} \begin{bmatrix} 1 & -5 & 3 \\ 1 & 1 & -1 \\ 0 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & -5/2 & 3/2 \\ 1/2 & 1/2 & -1/2 \\ 0 & 2 & -1 \end{bmatrix}$$

Now, $AX = B$

$$\Rightarrow X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1/2 & -5/2 & 3/2 \\ 1/2 & 1/2 & -1/2 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \\ 7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 - \frac{25}{2} + \frac{21}{2} \\ 3 + 5/2 - 7/2 \\ 0 + 10 - 7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{6 - 25 + 21}{2} \\ \frac{6 + 5 - 7}{2} \\ 3 \end{bmatrix}$$

(5)

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\therefore x=1, y=2, z=3.$$

which is the reqd. soln of the given eqn. #

Home Work:-

(ii) The given system of equations are -

$$3x + y + z = 1.$$

$$2x + 5y + 7z = 52$$

$$2x - y + z = 4.$$

The above equation can be written in matrix form as -

$$\text{where, } A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & -1 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \& \quad B = \begin{bmatrix} 1 \\ 52 \\ 4 \end{bmatrix}$$

$$\text{Now, } |A| = \begin{vmatrix} 3 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & -1 & 1 \end{vmatrix}$$

$$= 3(5+7) - 1(2-14) + 1(-2-10)$$

$$= 3 \cdot 12 - 1(-12) + 1(-12)$$

$$= 36 + 12 - 12$$

$$= 36.$$

$$\neq 0.$$

Since $|A| \neq 0$, the given equation has unique soln.

Let us find A^{-1} .

1st row :- co-factor of 3 = $\begin{vmatrix} 5 & 7 \\ -1 & 1 \end{vmatrix} = 5+7 = 12$

" " 1 = $-\begin{vmatrix} 2 & 7 \\ 2 & 1 \end{vmatrix} = -(2-14) = 12.$

" " 1 = $\begin{vmatrix} 2 & 5 \\ 2 & -1 \end{vmatrix} = 2-2-10 = -12$

P.T.A.

$$= \begin{bmatrix} 01/36 & -1/18 & -1/3 \\ 1/3 & 1/36 & 5/36 \\ 13/36 & -19/36 & 13/36 \end{bmatrix}$$

$$= \begin{bmatrix} 01/36 & -2/36 & -12/36 \\ 12/36 & 1/36 & 5/36 \\ 19/36 & -19/36 & 13/36 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{adj}A = \frac{1}{36} \begin{bmatrix} 12 & -2 & 2 \\ 12 & 1 & -19 \\ -12 & 5 & 13 \end{bmatrix}$$

$$\therefore \text{adj}A = \begin{bmatrix} 12 & -2 & 2 \\ 12 & 1 & -19 \\ -12 & 5 & 13 \end{bmatrix}$$

$$\omega\text{-factor matrix of } A = \begin{bmatrix} 12 & 12 & -12 \\ -2 & 1 & 5 \\ 2 & -19 & 13 \end{bmatrix}$$

3rd row: $\omega\text{-factor of } 2 = \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = 1 = \begin{vmatrix} 2 & 5 \\ 2 & 5 \end{vmatrix} = 15 - 2 = 13$

" " $\omega\text{-factor of } 2 = \begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix} = -1 = \begin{vmatrix} 2 & 7 \\ 2 & 7 \end{vmatrix} = (21 - 2) = -2$

2nd row: $\omega\text{-factor of } 2 = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2 = \begin{vmatrix} 5 & 7 \\ 1 & 1 \end{vmatrix} = 7 - 5 = 2$

" " $\omega\text{-factor of } 2 = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = 2 = \begin{vmatrix} 2 & -1 \\ 2 & -1 \end{vmatrix} = 2 - 2 = 0$

" " $\omega\text{-factor of } 2 = \begin{vmatrix} 1 & 5 \\ 3 & 1 \end{vmatrix} = 5 = \begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} = 2 - 2 = 0$

1st row: $\omega\text{-factor of } 2 = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = -2 = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = (1+1) = 2$

⑥