N.3) For any $a, b, c \in \mathbb{Z}$, we have,

$$a \odot (b \oplus c) = a \odot (b+c-1)$$
$$= a+(b+c-1) - a(b+c-1)$$
$$= a+b+c-1 - ab - ca + a$$
$$= 2a + b + c - ab - ac - 1.$$

Again,
$$a \odot b \oplus a \odot c = (a+b-ab) \oplus (a+c-ac)$$
$$= (a+b-ab) + (a+c-ac) - 1.$$
$$= 2a + b + c - ab - ac - 1$$

$\therefore$ $a \odot (b \oplus c) = a \odot b \oplus a \odot c.$

Similarly, we can show that
$$b \oplus c \odot a = b \odot a \oplus c \odot a$$

Hence $(\mathbb{Z}, \oplus, \odot)$ is a ring.

Again we have,
$$a \odot b = a + b - ab$$
$$= b + a - ba$$
$$= b \odot a.$$

Hence $a \odot b = b \odot a$, $\forall a, b \in \mathbb{Z}$.

This shows that commutative law hold good in $\mathbb{Z}$.

Also we have,

$$a \odot 0 = a + 0 - a0$$
$$= a$$

$$\& \; 0 \odot a = 0 + a - 0a$$
$$= a$$

$\therefore$ $a \odot 0 = 0 \odot a = a$

This shows that '0' is the multiplicative identity of $\mathbb{Z}$.

Hence $(\mathbb{Z}, \oplus, \odot)$ is a commutative ring with unity.

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Theorem: In a ring R, prove that,

'09' (i) $a0 = 0a = 0$

,, (ii) $a(-b) = (-a)b = -ab$.

,, '09' (iii) $(-a)(-b) = ab$

(iv) $a(b-c) = ab - ac$, $\forall a, b, c \in R$.

Proof: (i) We have,

$$0+0 = 0$$
$$\Rightarrow a(0+0) = a0$$
$$\Rightarrow a0 + a0 = a0 \text{ (by distributive law)}$$
$$\Rightarrow a0 + a0 = a0 + 0 \quad [\because x+0 = 0+x = x]$$
$$\Rightarrow a0 = 0 \text{ (by left cancellation law)}$$

Again,

$$0+0 = 0$$
$$\Rightarrow (0+0)a = 0a$$
$$\Rightarrow 0a + 0a = 0a \text{ (by distributive law)}.$$
$$\Rightarrow 0a + 0a = 0a + 0$$
$$\Rightarrow 0a = 0 \text{ (by left cancellation law)}.$$

$\therefore a0 = 0a = 0$ Proved

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(ii) We have,

$$(-b) + b = 0$$
$$\Rightarrow a(-b+b) = a0$$
$$\Rightarrow a(-b) + ab = 0 \quad [\because a0 = 0.a = 0]$$

$\Rightarrow a(-b)$ is the additive inverse of ab ie.

$$a(-b) = -(ab) \longrightarrow (i)$$

$x + x_1 = 0$
$\Rightarrow x_1$ is the additive inverse of x ie
$x_1 = -x_1$

Again, $(-a) + a = 0$

$$\Rightarrow ((-a)+a)b = 0b$$
$$\Rightarrow (-a)b + ab = 0$$

$\Rightarrow (-a)b$ is the additive inverse of ab ie.

$$(-a)b = -(ab) \longrightarrow (ii)$$

from (i) & (ii) we get —

$$a(-b) = (-a)b = -(ab)$$

Proved

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(iii) We know that,

$$a(-b) = (-a)b = -(ab)$$

We now have,

$$(-a)(-b) = -(a(-b))$$
$$= -(-(ab))$$
$$= ab$$

$$\therefore (-a)(-b) = ab$$

(iv) $a(b-c) = a(b+(-c))$

$= ab + a(-c)$ (by distributive law)

$= ab + (-ac)$ $[\because a(-b) = -(ab)]$

$= ab - ac$

$\therefore a(b-c) = ab - ac, \ \forall a, b, c \in R$ proved

10) Q:→ Define ring with zero divisor and give an example with justification.

Soln:→ Ring with zero divisor :→ A ring R is said to be a ring with zero divisor if $\exists\ a \neq 0,\ b \neq 0 \in R$ such $ab = 0$.

e.g.:- The set $M_{2\times2}$ of all 2×2 matrices over reals is a ring with zero divisor. for,

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 4 & 0 \end{pmatrix} \in M_{2\times2} \text{ but}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 4 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

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