Q:→ Define ring without zero divisor and give an example with justification.

Soln:→ Ring without zero divisor:→ A ring R is without zero divisor if $ab=0 \Rightarrow a=0$ or $b=0$, $\forall a,b \in R$.

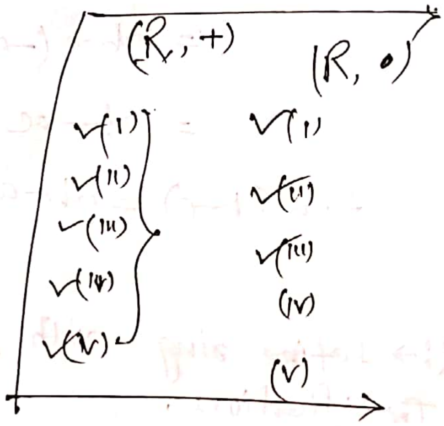
e.g → The set of integers $\mathbb{Z}$ is a ring without zero divisor. For,

$$ab=0 \Rightarrow a=0 \text{ or } b=0, \ \forall a,b \in \mathbb{Z}.$$

Some Important Definition:→

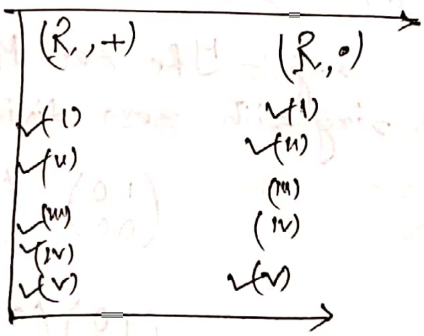
1. Ring with unity:→ A ring R is said to be a ring with unity if it posseses multiplicatives identity ie. $x.1=1.x=x, \forall x \in R$

e.g:→ The set of integers $\mathbb{Z}$ is a ring with unity.



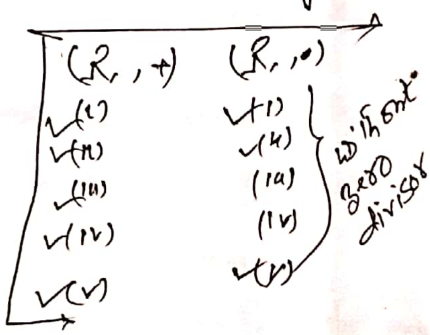
2. Commutative Ring:- A ring R is said to be a commutative ring if it satisfies commutative law w.r.to multiplication ie, $xy=yx, \ \forall x,y \in R$.

e.g:- The set of integer $\mathbb{Z}$ is a commutative ring.



3. Integral domain:→ A commutative ring R is said to be an integral domain if it is without zero divisor.

eg:- The set of integer $\mathbb{Z}$ is an integral domain.



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4. Division Ring :→ A ring R with unity is said to be a division ring if every non-zero element of it has its multiplicative inverse.

e.g. → The set of real number $\mathbb{R}$ is a division ring.

(R, +)	(R, ·)
✓(i)	✓(i)
✓(ii)	✓(ii)
✓(iii)	✓(iii)
✓(iv)	✓(iv)
✓(v)	(v)

5. Field :→ A commutative ring R with unity is said to be a field if every non-zero element of it has its multiplicative inverse.

e.g. :- The set of real number $\mathbb{R}$, ~~is a~~ the set of complex number $\mathbb{C}$ are fields.

(R, +)	(R, ·)
✓(i)	✓(i)
✓(ii)	✓(ii)
✓(iii)	✓(iii)
✓(iv)	✓(iv)
✓(v)	✓(v)

Q :→ What is division ring? What is the difference between division ring and field?

Ans :→ Division ring :- see above.

2nd part :→ A field posseses commutative law w.r. to multiplication but a division ring can not satisfies commutative law w.r. to multiplication.

2018 Q :→ Define integral domain and field? Give an example with justification of a field.

2012 Q :→ Give an example of

(i) A commutative ring with unity
(ii) A commutative ring without unity
(iii) A non-commutative ring with unity
(iv) A non-commutative ring without unity.

Sol^n :→ (i) The set of integer $\mathbb{Z}$ is a commutative ring with unity.

(ii) The set of even integers $\mathbb{E}$ is a commutative ring without unity.

(iii) The set of ~~2x2~~ ~~matrices~~ $M_{2\times 2}$ of all matrices is a non-commutative ring with unity.

Date 02/01/10

07 Q:→ In a ring R, $x^2 = x$ for all $x \in R$, show that

(i) $x+x=0$ or $2x=0$

(ii) $x+y=0 \Rightarrow x=y$

(iii) $xy=yx$ or R is commutative.

Proof:→ Given that,

$$x^2 = x, \ \forall x \in R \quad \longleftarrow (1)$$

(i) $\because x \in R$

$\therefore x, x \in R$

$\Rightarrow x+x \in R$

$\Rightarrow (x+x)^2 = x+x$ (using (1))

$\Rightarrow (x+x)(x+x) = x+x$

$\Rightarrow x(x+x) + x(x+x) = x+x$

$\Rightarrow x^2 + x^2 + x^2 + x^2 = x+x$

$\Rightarrow x+x+x+x = x+x$

$\Rightarrow (x+x)+(x+x) = (x+x)+0$

$\Rightarrow x+x = 0$ (by left cancellation law)

$\Rightarrow 2x = 0$ Proved