$\therefore\ a = \dfrac{9x_3 - 2x_1 + 4x_2 - 6x_3}{9}$

$= \dfrac{3x_3 - 2x_1 + 4x_2}{9}$

$\therefore (2) \Rightarrow b = x_2 + c - 2a$

$= x_2 + \cancel{x_3} - \frac{2}{9}(x_1 - 2x_2 + 3x_3) - 2 \cdot \dfrac{3x_3 - 2x_1 + 4x_2}{9}$

$= x_2 + \cancel{x_3} - \dfrac{2x_1 - 4x_2 + 6x_3}{9} - \dfrac{6x_3 - 4x_1 + 8x_2}{9}$

$= \dfrac{9x_2 + \cancel{9x_3} - 2x_1 + 4x_2 - 6x_3 - 6x_3 + 4x_1 - 8x_2}{9}$

$= \dfrac{5x_1 - x_2 - 3x_3}{9}$

Thus for any $x = (x_1, x_2, x_3) \in \mathbb{R}^3(\mathbb{R})$,
$\exists\ a = \dfrac{3x_3 - 2x_1 + 4x_2}{9}$, $b = \dfrac{5x_1 - x_2 - 3x_2}{9}$, $c = \dfrac{x_1 - 2x_2 + 3x_3}{9} \in \mathbb{R}$
such that $x = a\alpha_1 + b\alpha_2 + c\alpha_3$

This shows that $L(S) = \mathbb{R}^3(\mathbb{R})$.

Home Work
H.W (See pg.no-95) Hence S is a basis of $\mathbb{R}^3(\mathbb{R})$.

—o—

07 Q(1) Show that the vector $(1,0,2)$ & $(2,1,4)$ are linearly independent in $\mathbb{R}^3(\mathbb{R})$.

06 0.4 Q(2) In the vector space $\mathbb{R}^3(\mathbb{R})$, show that the vectors $(0,2,-4)$, $(1,-2,-1)$ & $(1,-4,3)$ are linearly dependent and the vectors $(1,2,0)$, $(0,3,1)$ & $(-1,0,1)$ are L.I.

06 Q(3) Show that the vectors $(1,2,1)$, $(2,1,0)$ & $(1,-1,2)$ form a basis of $\mathbb{R}^3(\mathbb{R})$.

04, 06 Q.(4) Show that the set of vectors $\alpha_1 = (2,1,4)$, $\alpha_2 = (-3,2,-1)$, $\alpha_3 = (1,-3,-2)$ in $\mathbb{R}^3(\mathbb{R})$ is L.I.

05 Q.(5) Prove that the vectors $(0,1,-2)$, $(1,-1,1)$, $(1,2,1)$ forms a L.I. set. in $\mathbb{R}^3(\mathbb{R})$

05 Q.(6) Show that the set $\{(1,0,0), (1,1,0), (1,1,1), (0,1,0)\}$ is not a basis of $\mathbb{R}^3(\mathbb{R})$.

04 Q.(7) Examine if $\{(1,0,0), (1,1,0), (1,1,1)\}$ forms a basis for $\mathbb{R}^3(\mathbb{R})$.

Q.(8). Examine if $\{(1,1,1), (1,2,3), (2,-1,1)\}$ forms a basis of $\mathbb{R}^3(\mathbb{R})$.

—o—

Imp. Q. 5 Dt. 27.9.08

Q. Show that the vectors (x_1, y_1) & (x_2, y_2) in $\mathbb{R}^2(\mathbb{R})$ is L.D. if an only if $x_1y_2 = x_2y_1$.

Solⁿ Let. $S = \{(x_1, y_1), (x_2, y_2)\}$

First let. S be linearly dependent. Then $\exists\ \alpha_1, \alpha_2 \in \mathbb{R}$ not all zero such that -

$$\alpha_1(x_1, y_1) + \alpha_2(x_2, y_2) = (0,0) \longrightarrow (1)$$

Let, $\alpha_1 \neq 0$, Then α_1^{-1} exists $\alpha_1\alpha_1^{-1} = \alpha_1^{-1}\alpha_1 = 1$

Then from (1) we get -

$$\alpha_1^{-1}\alpha_1(x_1, y_1) + \alpha_1^{-1}\alpha_2(x_2, y_2) = (0,0)$$

$$\Rightarrow 1(x_1, y_1) + \alpha_1^{-1}\alpha_2(x_2, y_2) = (0,0)$$

$$\Rightarrow (x_1 + \alpha_1^{-1}\alpha_2 x_2\ ,\ y_1 + \alpha_1^{-1}\alpha_2 y_2) = (0,0)$$

$$\therefore x_1 + \alpha_1^{-1}\alpha_2 x_2 = 0 \quad \& \quad y_1 + \alpha_1^{-1}\alpha_2 y_2 = 0$$

$$\Rightarrow x_1 = -\alpha_1^{-1}\alpha_2 x_2 \qquad \Rightarrow y_1 = -\alpha_1^{-1}\alpha_2 y_2$$

$$\Rightarrow \frac{x_1}{x_2} = -\alpha_1^{-1}\alpha_2 \qquad \Rightarrow \frac{y_1}{y_2} = -\alpha_1^{-1}\alpha_2$$

$$\therefore \frac{x_1}{x_2} = \frac{y_1}{y_2}$$

$$\Rightarrow x_1 y_2 = y_1 x_2$$

Conversely, Let, $x_1 y_2 = x_2 y_1 \longrightarrow (1)$

We now show that ~~if $x_1 = y_1 = 0$~~ S is L.D.

We observe that if $x_1 = y_1 = 0$ or $x_2 = y_2 = 0$.

Then S is L.D. being a set containing zero vector (0,0).

Let, $x_1 \neq 0$. Then $\frac{x_2}{x_1} \in \mathbb{R}$

We now have, $\left(\frac{-x_2}{x_1}\right)(x_1, y_1) + 1(x_2, y_2) = \left(\frac{-x_1x_2}{x_1}, \frac{-y_1x_2}{x_1}\right) + (x_2, y_2)$

$$= \left(-x_2, -\frac{x_1y_2}{x_1}\right) + (x_2, y_2)$$

$$= (-x_2, -y_2) + (x_2, y_2)$$

$$= (0,0)$$

This shows that S is L.D.

Q. Show that the subset $S=\{x_1, x_2, x_3, x_4\}$ of the vector space $M(\mathbb{R})$ of 2×2 matrices is a basis of $M(\mathbb{R})$, where -

02 A

$$x_1=\begin{bmatrix}1&0\\0&0\end{bmatrix},\ x_2=\begin{bmatrix}0&1\\0&0\end{bmatrix},\ x_3=\begin{bmatrix}0&0\\1&0\end{bmatrix},\ x_4=\begin{bmatrix}0&0\\0&1\end{bmatrix}$$

What is the dimension of $M(\mathbb{R})$.

Sol^n To show S is L.I. :-

Let $\alpha_1, \alpha_2, \alpha_3, \alpha_4 \in \mathbb{R}$ such that

$$\alpha_1 x_1+\alpha_2 x_2+\alpha_3 x_3+\alpha_4 x_4=\bar{0}$$

$$\Rightarrow \alpha_1\begin{bmatrix}1&0\\0&0\end{bmatrix}+\alpha_2\begin{bmatrix}0&1\\0&0\end{bmatrix}+\alpha_3\begin{bmatrix}0&0\\1&0\end{bmatrix}+\alpha_4\begin{bmatrix}0&0\\0&1\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}\alpha_1&0\\0&0\end{bmatrix}+\begin{bmatrix}0&\alpha_2\\0&0\end{bmatrix}+\begin{bmatrix}0&0\\\alpha_3&0\end{bmatrix}+\begin{bmatrix}0&0\\0&\alpha_4\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}\alpha_1&\alpha_2\\\alpha_3&\alpha_4\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

$\therefore \alpha_1=0$

$\alpha_2=0$

$\alpha_3=0$

$\alpha_4=0$

$\therefore \alpha_1 x_1+\alpha_2 x_2+\alpha_3 x_3+\alpha_4 x_4=\bar{0}$

$\Rightarrow \alpha_1=\alpha_2=\alpha_3=\alpha_4=0$

$\therefore$ S is linearly independent in $M(\mathbb{R})$.

To show $L(S)=M(\mathbb{R})$:-

Let, $x=\begin{bmatrix}p&q\\r&s\end{bmatrix}$ be any element of M.

Thus $p, q, r, s\in\mathbb{R}$, we now have -

$$x=\begin{bmatrix}p&q\\r&s\end{bmatrix}$$

$$=p\begin{bmatrix}1&0\\0&0\end{bmatrix}+q\begin{bmatrix}0&1\\0&0\end{bmatrix}+r\begin{bmatrix}0&0\\1&0\end{bmatrix}+s\begin{bmatrix}0&0\\0&1\end{bmatrix}$$

$$=px_1+qx_2+rx_3+sx_4$$

This shows that $L(S)=M(\mathbb{R})$.

$\therefore$ S is a basis of $M(\mathbb{R})$

Hence, the dimension of $M(\mathbb{R})$ is 4.

P.T.O