This shows that A satisfies it's characteristic equation.
Hence, Cayley-Hamilton Theorem is varified.

2nd part,

We have, $A^2 - 5A + 7I = 0$

$$\Rightarrow A^2 \cdot A^{-1} - 5AA^{-1} + 7IA^{-1} = 0$$

$$\Rightarrow A - 5I + 7A^{-1} = 0$$

$$\Rightarrow 7A^{-1} = 5I - A$$

$$\Rightarrow 7A^{-1} = 5\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$$

$$\Rightarrow 7A^{-1} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} - \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$$

$$\Rightarrow 7A^{-1} = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{7}\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 2/7 & -1/7 \\ 1/7 & 3/7 \end{pmatrix} \quad \underline{\text{Ans:}}$$

07. H.W. Q. If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$ and hence show that $A^{-1} = \begin{pmatrix} -3 & 0 & 2 \\ -1 & \frac{1}{2} & \frac{1}{2} \\ 2 & 0 & -1 \end{pmatrix}$

01

Q. Using Cayley-Hamilton theorem, express $2A^5 - 3A^4 + A^2 - 4I$ as $138A - 403I$, where $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$.

Sol^n Here, $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$

$\therefore$ The characteristic equation of the matrix A is

$$|A - xI| = 0$$

$$\Rightarrow \begin{vmatrix} 3-x & 1 \\ -1 & 2-x \end{vmatrix} = 0$$

$$\Rightarrow (3-x)(2-x) + 1 = 0$$

$$\Rightarrow 6 - 5x + x^2 + 1 = 0$$

$$\Rightarrow x^2 - 5x + 7 = 0$$

According to Cayley-Hamilton theorem,

We have,

$$A^2-5A+7I=0 \longrightarrow ①$$

Now, $\Rightarrow A^2=5A-7I$

$$A^3=5A^2-7IA$$
$$=5A^2-7A$$
$$=5(5A-7I)-7A$$
$$=25A-35I-7A$$
$$=18A-35I$$

$$A^4=18A^2-35A$$
$$=18(5A-7I)-35A$$
$$=90A-126I-35A$$
$$=55A-126I$$

$$A^5=55A^2-126A$$
$$=55(5A-7I)-126A$$
$$=275A-385I-126A$$
$$=149A-385I$$

Now, $2A^5-3A^4+A^2-4I=2(149A-385I)-3(55A-126I)$
$+(5A-7I)-4I$

$$=298A-770I-165A+378I+5A-7I-4I$$
$$=138A-403I$$ Proved

(which is a linear polynomial in A)

H.W Q. Using Cayley-Hamilton theorem, express –
$A^5-4A^4-7A^3+11A^2-A-10I$ as a linear polynomial in A
where $A=\begin{pmatrix}1&4\\2&3\end{pmatrix}$

Sol^n: Here, $A=\begin{pmatrix}1&4\\2&3\end{pmatrix}$

$\therefore$ The charateristic equ^n. of the matrix A is

$$|A-xI|=0$$
$$\Rightarrow \begin{vmatrix}1-x & 4\\ 2 & 3-x\end{vmatrix}=0$$

P.T.O

$\Rightarrow (1-x)(3-x) - 8 = 0$

$\Rightarrow 3 - 4x + x^2 - 8 = 0$

$\Rightarrow x^2 - 4x - 5 = 0$

According to Cayley - Hamilton Theorem -

we have - $A^2 - 4A - 5I = 0$

$\Rightarrow A^2 = 4A + 5I \rightarrow (1)$

Now, $A^3 = 4A^2 + 5AI$

$= 4A^2 + 5A$

$= 4(4A + 5I) + 5A$

$= 21A + 20I$

$A^4 = 21A^2 + 20AI$

$= 21(4A + 5I) + 20A$

$= 84A + 105I + 20A$

$= 104A + 105I$

$A^5 = 104A^2 + 105AI$

$= 104(4A + 5I) + 105A$

$= 416A + 520I + 105A$

$= 521A + 520I$

Now, $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$

$= 521A + 520I - 4(104A + 105I) - 7(21A + 20I) +$
$11(4A + 5I) - A - 10I$

$= 621A + 520I - 416A - 420I - 147A - 140I + 44A$
$+ 55I - A - 10I$

$= A + 5I$

which is a linear polynomial in A.